

by Ádám Siska

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Introduction

Variations is a piece for piano or harpsichord¹ based on the evolution of a system with six different states. These states are mapped to one or more musical parameters desribed by *Rules* which are different for each variation. The *Algorithm* describing the evolution of the states is a combination of several Markov-tables based on the pitches of the Thema. Therefore the performer has to create her or his own realizations of these variations.

The Algorithm

The Thema uses six pitches, each one of them representing a state of the system:



To build the algorithm, first we 'quantize' the Thema using quavers. Here is the beginning as an example:



The first measure of the Thema (left) and the quantized form of the same measure (right).

Once done we have a series of states which we use to build Markov-tables: we need to create all tables from 0^{th} to 6^{th} order.

To get a new state, first we evaluate the next state of each separate Markov-table (using the last states of the system as the 'history' of each Markov-table) and we assign a probability weight to these 'new state candidates' proportional to the order of the corresponding Markov-table plus one (that is, 1 for the 0th order table, 2 for the 1st order table etc). Tables which doesn't have a next state for the current history (which can

¹For harpsichord some additional details are needed, see the notes at the end of the score.

happen with any table but the 0th order one²) are excluded. Finally, we choose the actual next state by choosing randomly a new state from the 'new state candidates' according to their assigned weights.

This algorithm is used in all variations except the 2^{nd} one, where the algorithm differs slightly. For details, see *Rules* for Variation II.

Rules

These rules describe the way to build each variation using the states given by the *Algorithm*.

Unless otherwise stated, repeating pitches should be tied to each other creating a longer note. If a rule doesn't use all of the six states, the *Algorithm* has to be modified so that unused states should be first removed from the **Thema** and then the rest of the *Algorithm* has to be applied. If a *repetition filtered* criterion appears, the sequence of states mustn't have repetitions. To reach this, the tables used by the *Algorithm* must be re-generated with a modified transcription of the **Thema** where every repetition has been priorly removed and the new states for each step must be generated until the new state differs from the last one³.

Variation I

Five voices, each with an own sequence of states has to be created. These states should be assigned to pitches according to the mappings listed at the end of the section. The duration – measured in crotchets – of each voice is as follows:

	Voice I	Voice II	Voice III	Voice IV	Voice V
Begin	1	2	34	53	66
End	33	77	65	77	77

The last 4 crotchets in Voices 2 and 4 should be played an octave higher. If there was a long note at the breaking point, it has to be split. In Voices 3 and 5 repetitions must be filtered. An *accelerando* starts at the 34th and a *molto accelerando* at the 66th crotchet.

The mapping of the states to the pitches is

²Note that the 0th order table is actually no more than a simple probability distribution (without any history) of the states.

³If *repetitions filtered* appears only partially, like in Variation II, where only some specific chords have to be filtered, the state sequences use the original tables, the filtering of the repetitions means that new states must be generated until the new one differs from the last one.



Variation II

There are five sections in this variation using a modified version of the *Algorithm*. The number of simultaneously used states changes from section to section, the only modification of the algorithm is that instead of using all the 7 tables from 0^{th} to 6^{th} order, each section uses only as many tables as the number of used states in that section (for example, the section with states 0 - 3 is using the tables between 0^{th} and 3^{rd} order).

The variation consists of chords of equal durations (quavers). The repeating pitches (chords) shouldn't be tied to each other. The first chord is which is followed by five sections lasting 240, 150, 90, 45 and 30 quavers, respectively. The mapping of the states for each section is as follows:





Variation III

This variation consists of 41 bars containing intervals, each one with independent and free duration (keep in mind that the character of the variation has to be slow) but fixed rythm patterns. There are two types of rythmic elements: • indicates a long note (at least a half note) and • a short one (from an acciaccatura up to a quaver). Intervals in the same bar must be played *legato* but a short break (like a short *fermata*) must be taken between bars. The first bar is $\underbrace{\$intervals}_{\$intervals}$ All the other bars are generated by four sequences generated by the *Algorythm* (defining the rythm patterns, the base pitches of the intervals themselves and the dynamics). Their mappings are:



Some additional details are needed to apply these rules:

- **Rythm** In case of the 2nd, 3rd and 5th states, repetitions must be filtered (regarding the intervals obtained by the following two rules).
- **Base pitch** The actual pitch has to be selected randomly (giving each pitch equal probability weight) from the pitches contained by the actual region.
- **Intervals** The mappings are: 0 Perfect Fourth; 1 Minor Second; 2 Major Seventh; 3 Perfect Unison; 4 Major Second; 5 Minor Seventh.

Variation IV

There are two voices, each one of them having two sequences of states generated by the *Algorithm*. The first sequence defines the rythmic pattern while the second one defines the pitches. The mappings are:



Pitch state 0 maps to rest for both voices. There are several subsequent sections where each voice uses the following ranges of states (for both state sequences pertaining to the given voice):

Duration	15	15	29	29	29	29	29	29	15	15
Voice I	0 - 2	0 - 2	0 - 3	0 - 4	0 - 5	0 - 4	0 - 3	0 - 2	0 - 1	0
Voice II	0	0 - 1	0 - 2	0 - 3	0 - 4	0 - 5	0 - 4	0 - 3	0 - 2	0 - 2

Duration values are measured in crotchets.

Variation V

There are two voices. The first one plays semiquavers (using *without repetition* constraint) with the following mapping:



The second one is a sequence of subsequent chords. The duration and the pitches of these chords are described by five state sequences (one for the duration, one for the pitch base of the chord and the other three describing the pitches themselves). The mappings are:

State	0	1	2	3	4	5
Duration	21	13	8	5	3	2
Pitch base	0	34	46	58	70	82
Pitch offset I	0	1	2	3	4	
Pitch offset II	4	5	6	7	8	
Pitch offset III	8	9	10	11	12	

Durations are measured in semiquavers. If the pitch base is 0, a rest should be played. Otherwise the three pitch offsets must be added to the pitch base value; the resulting numbers should be treated as MIDI pitch values.

The following table describes the length of each section (in semiquavers) and the state ranges used by each one of the sequences in each section:

Section length	105	130	120	100	281	200	75	105	75
Voice I	0 - 1	0 - 2	0 - 3	0 - 4	0 - 4	0 - 4	0 - 4	0 - 4	0 - 3
Chord duration	0 - 1	0 - 2	0 - 3	0 - 4	1 - 4	3 - 4	4 - 5	2 - 4	3 - 4
Pitch base	0 - 2	0 - 3	0 - 4	0 - 5	1 - 5	2 - 5	3 - 5	0 - 2	1

To get a state in a range where the low bound (s_{\min}) of the state range is not 0, a state must be generated in the range between 0 and $s_{\max} - s_{\min}$ (where s_{\max} is the high bound of the original range) and then s_{\min} has to be added to the result.

If a section ends in the middle of a chord, the section should be extended until the end of the chord.

The last chord of the section is 2 This chord must be played with the sustain pedal held. The sustain pedal must remain unreleased until the end of the next variation.

Variation VI

This variation has to be played with the sustain pedal held all time. The mapping of the states is



The variation consists of three sections. The first one lasts 233 quavers, the second one 377 and the last one 610. The first section uses only states 0 - 3, the second one 0 - 4 and the last section uses all the possible states.

Additional remarks

The Thema and all variations must be played *attacca*. The tempo should be the same for all variations except for Variation III where durations are free, for Variation V which should be played fast (but slower than double-tempo) and for Variation VI which should be played a bit slower.

If the piece is performed on harpsichord, some changes need to be done. In Variation I, the last four crotchets of Voices 2 and 4 should be played in the same octave as the

previous notes. In Variation II the chord should be played on a loudly registrated manual. Some of the mappings in Variation V change slightly as well:

State	0	1	2	3	4	5
Pitch base	0	34	45	56	67	78
Pitch offset II	3	4	5	6	7	
Pitch offset III	6	7	8	9	10	

Variation VI should be played an octave lower, the *laissez vibrer* should be simulated by holding the keys as long as possible and the last chord from Variation V must be held as long as possible as well. If was not present on the instrument, the Thema and variations I, II, IV and VI must be transposed accordingly.

Thema

